

Mathematics

Grade Span 9/10

The development of mathematical competencies is supported in specific ways through the introduction of Computer Algebra Systems (CAS). This calls for a continuous, mandatory use of CAS in the classroom. Parallel to this, students continuously develop basic competences through coursework performed without aids. *Without aids* means that no tools other than writing instruments may be used.

Arithmetic/Algebra - Working with Numbers, Variables and Symbols

Subject Matter Competencies

- convert among power, root and logarithmic notation
- justify the Laws of Exponents with examples and apply them without the use of aids
- convert angle measurements between degrees and radians
- convert terms and select and apply suitable methods for the determination of solutions to equations and inequalities
- graphically interpret solution sets of linear equation systems with two equations and two variables
- explain and apply an algorithmic solution method to linear systems of equations
- select and apply suitable methods for determining the solution sets of systems of equations
- investigate questions of solvability and solution multiplicity of quadratic equations and systems of linear equations
- apply a solution formula to the normal form of a quadratic equation
- without any aids in terms of content or calculation, solve:
 - linear systems of equations with two equations and two variables
 - simple quadratic equations
 - simple exponential equations
 - simple fractional equations
- apply knowledge of equations and systems of equations to problems drawn from everyday situations in mathematics, science, economics and technology

Methodological Competencies

- communicate and reflect upon mathematical solutions and results in an understandable and appropriate form:
 - in writing
 - through verbal explanation
 - in a presentation
- apply the interactive exploration capabilities of a CAS to experimental and heuristic work in pure and applied mathematics contexts

Social and Emotional Competencies

- independently develop and implement various solution plans and assess their advantages and disadvantages
- work cooperatively in groups on complex assignments
- understand the mathematical arguments of other students and evaluate their correctness
- and completeness
- critically assess and work with the results and instructions provided by the CAS and, if necessary, make appropriate changes to the student's solution strategy

Functions - Exploring, Describing and Representing Relationships and Changes

Subject Matter Competencies

- investigate and graphically represent quadratic functions on the basis of definition and range of values, vertex, axis intersection points, monotonicity and symmetry
- describe the influence of parameters on the properties and graphs of bell-shaped quadratic functions
- based on graphical representations of quadratic functions, determine the equation of the quadratic functions
- based on points on the graph of a function, determine the equation of a quadratic function
- describe and solve pure and applied mathematical problems using quadratic functions
- specify characteristic properties of:
 - power functions $f(x)=x^n$ with integer and rational exponents
 - exponential functions $f(x)=a^x$, $a \in \mathbb{Q}^+$
 - the sine function $f(x)=\sin(x)$ and the cosine function $f(x)=\cos(x)$

- describe the relationship between the graph of the functions $a \cdot f(x-d)+c$ and the graph of the function $f(x)$ and in addition the influence of the parameter b on the properties of sine functions $f(x)=\sin(bx)$ and cosine functions $f(x) = \cos(bx)$
- determine an equation for a function based on their representation as a possible function type (in simple cases without the use of aids)
- represent functions and describe their properties (in simple cases without aids)
- explain the relationship between functions and their inverses using examples
- explain the limit value based on given assumptions and describe graphs (also considering horizontal and vertical asymptotes) using the limit value representations:
 - $\lim_{x \rightarrow \pm\infty} f(x)$ and $\lim_{x \rightarrow x_0} f(x)$
- Use functions to solve pure and applied mathematical problems, including using an appropriate function to approximately describe the relationship between two data sets

Methodological Competencies

- extract, work with and interpret information from functional equations and computer displays
- use a formula table properly

Social and Emotional Competencies

- document and present one's findings regarding functional relationships using mathematical terminology in oral and written form
- use a CAS to check one's own results
- evaluate results critically

Geometry - Describing Two- and Three-Dimensional Figures According to Size and Shape

Subject Matter Competencies

- draw congruent two-dimensional figures using centric stretching with a positive scale factor
- describe the influence of the scale factor on the size of angles, the length of sides, area and volume
- illustrate centric stretching and congruence using dynamic geometry software
- for right triangles:
 - specify without the use of aids and illustrate using examples the definitions of sine, cosine and tangent of an angle

- calculate angles and side lengths using sine, cosine and tangent
- explain using examples and apply (in simple cases without aids) the main Similarity Theorem for triangles and the Ray Theorem (first and second part)
- apply the Sine Theorem and the Cosine Theorem to calculate side lengths and angles
- state and apply the formula $\text{Area} = \frac{1}{2}a \cdot b \cdot \sin(\gamma)$ to given triangles
- properly decompose and use in calculations scale drawings and sketches of composite three-dimensional bodies
- calculate the surface area and volume of composite bodies

Methodological Competencies

- apply solution strategies to geometric constructions and calculations:
 - decompose a problem into sub-problems
 - recognize special lines, triangles and polygons in two- and three-dimensional figures
 - work through problems forward and backward
- present, explain and reflect upon solutions and results in an understandable and appropriate manner

Social and Emotional Competencies

- plan work steps for individual and cooperative work and implement them independently
- compare and evaluate solutions, arguments and representations

Probability - Working with Data and Chance

Subject Matter Competencies

- determine and describe the probabilities of related events using symbols such as: $A \cup B$, $A \cap B$, \bar{A}
- using four field tables and tree diagrams:
 - illustrate probability experiments having two or more stages
 - calculate the probability of events (in simple cases without aids)
 - examine events in simple contexts for probabilistic independence
- determine probability distributions of discrete random variables
- calculate and interpret the expected value and standard deviation of discrete random variables (in simple cases without aids)
- calculate using the symbol Σ
- describe Bernoulli chains as multistage random experiments and use the Bernoulli formula

- to determine binomial coefficients and interpret them
- evaluate conditions for the applicability of the Bernoulli formula and critically evaluate the results

Methodological Competencies

- evaluate and display data, including by using computer software, in tables and histograms
- formulate, evaluate and present Ideas for and the results of the description, simulation and calculation of experiments in probability
- graphically represent the probability distributions of discrete random variables
- interpret graphical representations of the probability distributions of discrete random variables

Social and Emotional Competencies

- evaluate results of probability calculations for plausibility and evaluate them critically
- assess opportunities and risks in stochastic contexts